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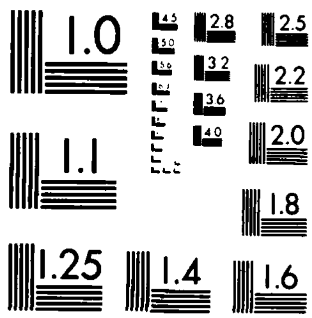

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

Hans-Eberhard Peters

October 1982

Thesis Advisor: J.D.Esary

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Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be

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Implementation of a Reliability Shorthand on the TI-59 Handheld Calculator

by

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from the

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It is shown how a reliability shorthand can be implemented on a handheld calculator.

Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be used to compute the reliability of a system.

Two TI-59 programs are provided as a computational aid.

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I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that T, the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution. In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to

failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.

II. THE CONCEPT OF A RELIABILITY SHORTHAND

A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

T : life length

$F(t) = P(T \leq t)$ be the distribution function of

Then

$$\bar{F}(t) = P(T>t)$$
$$= 1-F(t)$$

is the survival function of T.

In the case of the exponential distribution, $\bar{F}(t) = e^{-\lambda t}$, where λ is the failure rate. Translated into shorthand, the life distribution is denoted

$$EXP(\lambda).$$

B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time t. Let

T_1, T_2 : independent life lengths

$\bar{F}_1(t), \bar{F}_2(t)$: the corresponding survival functions

$f_1(t), f_2(t)$: the corresponding density functions

$T = T_1 + T_2$: the total life length

Then

$$\bar{F}(t) = P(T>t)$$
$$= P(T_1 + T_2 > t)$$
$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s)\, ds.$$

This means that T will exceed a specified time t when

-either T_1 exceeds t

-or T_1 is smaller than t, say equal to s, and T_2 exceeds t-s.

Integration with respect to s (i.e. summing over all possible values of s) is called the convolution of T_1 and T_2. When T_1 and T_2 are both exponentially distributed with failure rates λ_1 and λ_2 , i.e.

$$\bar{F}_1(t) = e^{-\lambda_1 t}$$
$$\bar{F}_2(t) = e^{-\lambda_2 t},$$

then the survival function of T is

$$\bar{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s} \quad ds.$$

Translated into shorthand, the survival function is denoted

$$EXP(\lambda_1) + EXP(\lambda_2).$$

This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with $EXP(\lambda_1)$ and $Exp(\lambda_2)$ lives respectively. From component 1 the system has an $EXP(\lambda_1)$ life to begin with. When component 1 fails, the system has an extra $EXP(\lambda_2)$ life.

C. MIXTURE OF DISTRIBUTIONS

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2 .$$

Now consider

$$T = \begin{cases} T_1 & \text{with probability } p_1 \\ T_2 & \text{with probability } p_2 \end{cases}$$

where $p_1 + p_2 = 1$.

Let D_1 and D_2 be the probability distributions of the random variables T_1 and T_2 respectively. The corresponding survival functions are $\overline{F}_1(t)$ and $\overline{F}_2(t)$.

Then

$$\overline{F}(t) = p_1 \overline{F}_1(t) + p_2 \overline{F}_2(t) .$$

In shorthand, the mixture of distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2 is denoted

$$MIX[\ p_1 D_1\ ,\ p_2 D_2\ ].$$

2. <u>Distributive Law</u>

Now let

$$T = T_3 + T'$$

where

$$T' = \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

Then

$$T = T_3 + \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

$$T = \begin{cases} T_3 + T_1 \quad \text{with probability } p \\ T_3 + T_2 \quad \text{with probability } 1-p. \end{cases}$$

The distributive law holds due to the fact that the sum of the mixing probabilities for T_1 and T_2 is equal to one.

The survival function of T can be found by convolution:

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t (p\bar{F}_1(t-s) + (1-p)\bar{F}_2(t-s)) f_3(s) ds.$$

With D_1, D_2, D_3 being the probability distributions for T_1, T_2, T_3, the distributive law can be applied to the shorthand notation:

$$D_3 + \text{MIX}[pD_1, (1-p)D_2] = \text{MIX}[\ p((D_1 + D_3), \\ (1-p)(D_2 + D_3)\ ].$$

Graphically this can be represented as follows:

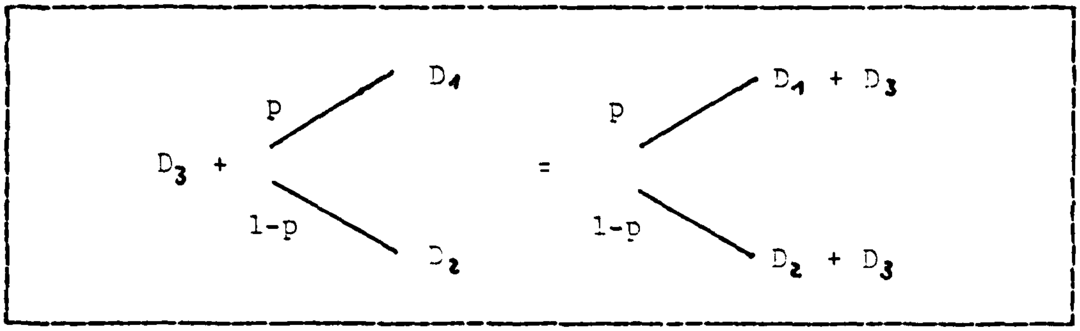


Figure 1: Distributive Property of the MIX-Notation

3. Degeneracy at the Origin

Let

$$P(T=0) = 1.$$

Then the distribution of T is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let $T = T_1 + T_0$

where T_1 and T_0 have probability distributions D_1 and ZERO and survival functions $\bar{F}_1(t)$ and $\bar{F}_0(t)$ respectively.

Then

$$\begin{aligned}\bar{F}(t) &= \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s) f_1(s)\,ds \\ &= \bar{F}_1(t).\end{aligned}$$

The ZERO-distribution doesn't add anything to another distribution, so for instance

$$D_1 + ZERO = D_1$$

$$D_2 + MIX[pD_1, (1-p)ZERO] = MIX[p(D_1 + D_2), (1-p)D_2].$$

III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig.2 .

Let

T : life of the system

T_1: life of component 1

T_2: life of component 2

$\bar{F}_1(t)$ = survival function of component 1

$= e^{-\lambda_1 t}$

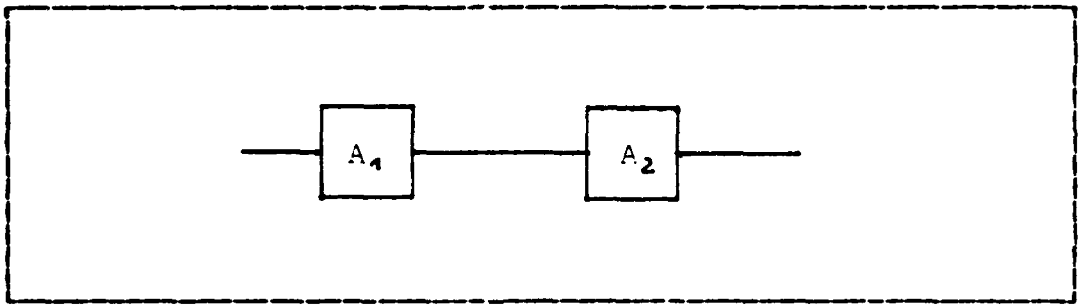


Figure 2: Two-Component Series System

$\bar{F}_2(t)$ = survival function of component 2

$= e^{-\lambda_2 t}$.

Then

$T = \min(T_1, T_2)$

$\bar{F}(t)$ = survival function of the system

$= P(\min(T_1, T_2) > t)$

$= P(T_1 > t, T_2 > t)$

Assuming independence of the two components

$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t)$

$= \bar{F}_1(t)\ \bar{F}_2(t)$

$= e^{-\lambda_1 t}\ e^{-\lambda_2 t}$

$= e^{-(\lambda_1 + \lambda_2) t}$.

The shorthand notation for this system is

EXP $(\lambda_1 + \lambda_2)$.

This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$.

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

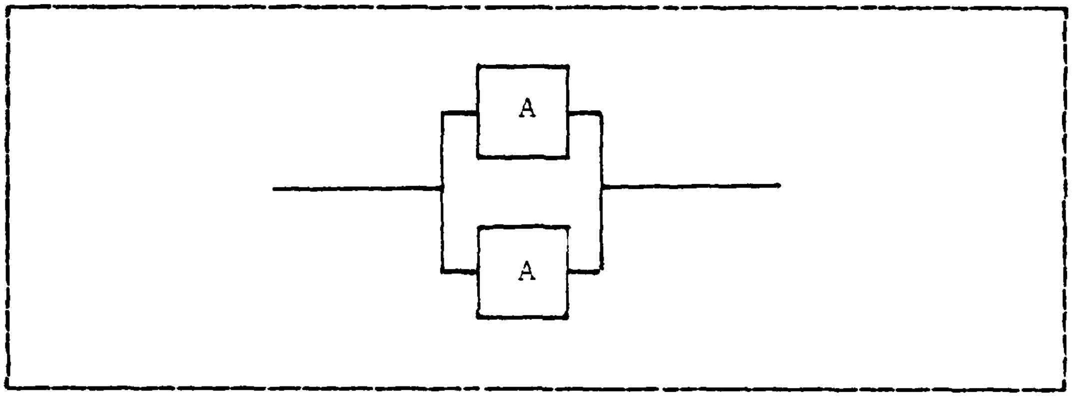


Figure 3: Two-Component Parallel System

Let

$$T_1 \sim EXP(\lambda), \quad T_2 \sim EXP(\lambda) .$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned} \overline{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1 - P(\max(T_1, T_2) \leqslant t) \\ &= 1 - P(T_1 \leqslant t, T_2 \leqslant t) \end{aligned}$$

Assuming independence of the two components,

$$\bar{F}(t) = 1 - P(T_1 \leq t)\ P(T_2 \leq t)$$

$$= 1 - F_1(t)\ F_2(t)$$

$$= 1 - (1-e^{-\lambda t})(1-e^{-\lambda t})$$

$$= 1 - (1-2e^{-\lambda t} + e^{-2\lambda t})$$

$$= 2e^{-\lambda t} - e^{-2\lambda t}.$$

The shorthand notation for the system is

$$EXP(2\lambda) + EXP(\lambda).$$

This follows intuition as the system has an $EXP(2\lambda)$ life to begin with and when one component fails it has an extra $EXP(\lambda)$ life due to the memoryless property of the exponential distribution.

3. Standby-System with Dissimilar Components

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be $T_1 \sim EXP(\lambda_1)$ and $T_2 \sim EXP(\lambda_2)$ respectively. Then the system time to failure is

$$T = T_1 + T_2$$

and the survival function of the system is

$$\bar{F}(t) = P(T > t)$$

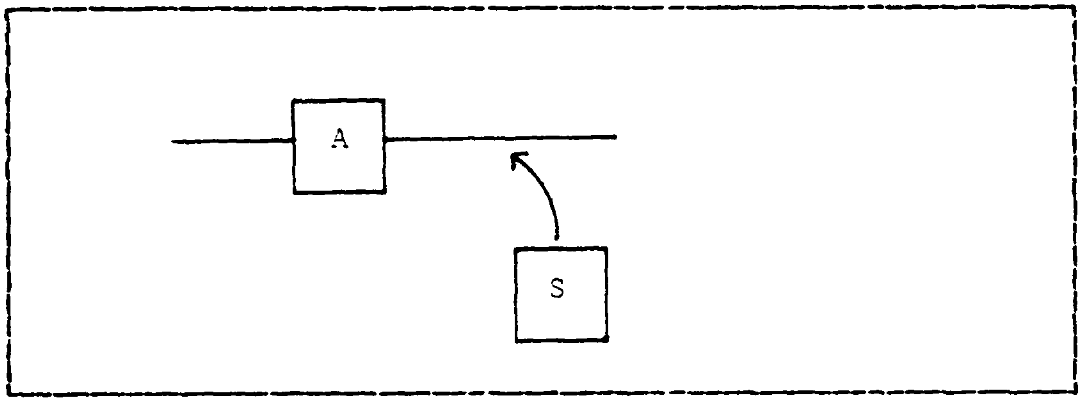


Figure 4: Standby System

$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s)\, ds$$

$$= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s}\, ds$$

$$= \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t}$$

The shorthand notation for the system's survival function should be obvious. The system has an EXP(λ_1) life from the active component and an additional EXP(λ_2) life from the spare. So the shorthand notation is

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).$$

B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2 . We now look at the case where

$$T_1 \sim EXP(\lambda_1) \text{ and } T_2 \sim EXP(\lambda_2).$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned}\bar{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1-P(\max(T_1, T_2) \leqslant t) \\ &= 1-P(T_1 \leqslant t, T_2 \leqslant t)\end{aligned}$$

Assuming independence of the two components

$$\begin{aligned}\bar{F}(t) &= 1-P(T_1 \leqslant t)\ P(T_2 \leqslant t) \\ &= 1-F_1(t)\ F_2(t) \\ &= 1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t}) \\ &= 1-(1-e^{-\lambda_1 t}-e^{-\lambda_2 t}+e^{-(\lambda_1+\lambda_2)t}) \\ &= e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-(\lambda_1+\lambda_2)t}.\end{aligned}$$

To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

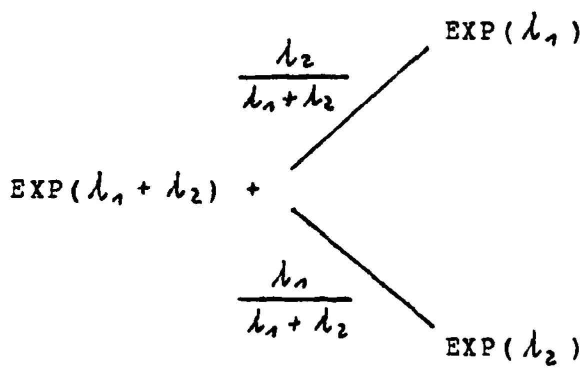
-either both components survive

-or component 1 fails and component 2 survives

-or component 2 fails and component 1 survives.

If one component fails and one survives, in $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ fraction of the cases the survivor will be component 1 and in $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ fraction of the cases it will be component 2.

This can graphically be represented as



Making use of the MIX-notation the shorthand notation then is

$$\mathrm{EXP}(\lambda_1 + \lambda_2) + \mathrm{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_1), \frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_2)]$$

and using the distributive property it becomes

$$\mathrm{MIX}[\frac{\lambda_2}{\lambda_1 + \lambda_2}(\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_1 + \lambda_2)), \\ \frac{\lambda_1}{\lambda_1 + \lambda_2}(\mathrm{EXP}(\lambda_2) + \mathrm{EXP}(\lambda_1 + \lambda_2))].$$

As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

$$\bar{F}(t) = \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_1 e^{-\lambda_1 s}\,ds\right)$$

$$+ \frac{\lambda_1}{\lambda_1+\lambda_2}\left(e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_2 e^{-\lambda_2 s}\,ds\right)$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} .$$

This verifies that the shorthand notation indeed represents the system's survival function.

2. Series System with One Spare

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate λ_1 and component 2 and the spare have the constant failure rate λ_2. The spare can only replace component 2.

Let

$\bar{F}_1(t)$: the survival function of component 1

$\bar{F}_2(t)$: the survival function of the standby system component 2 with its spare.

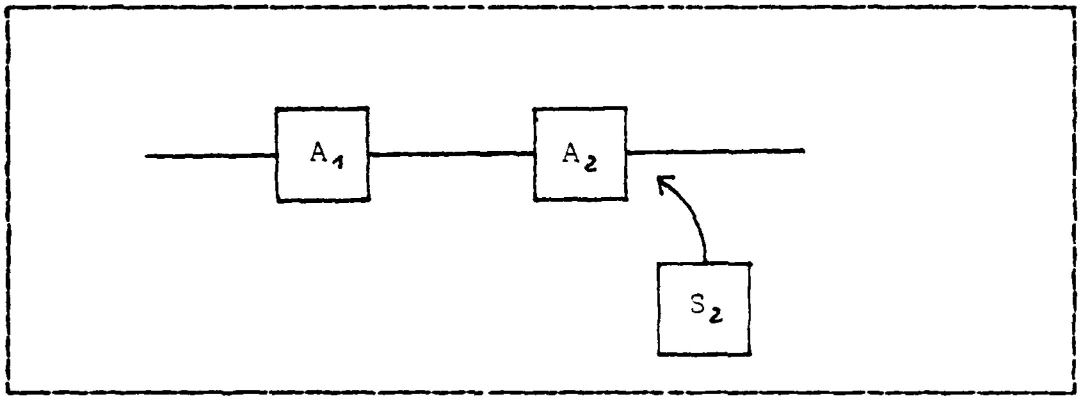


Figure 5: Series System with one Spare

The survival function for a standby system was derived in Chapter II.B. Therefore

$$\bar{F}_2(t) = e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_2 e^{-\lambda_2 s} ds$$

$$= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t ds$$

$$= (1+\lambda_2 t) e^{-\lambda_2 t} .$$

Now $\bar{F}_1(t) = e^{-\lambda_1 t}$

Then $\bar{F}(t) = \bar{F}_1(t)\ \bar{F}_2(t)$

$$= (1+\lambda_2 t) e^{-(\lambda_1+\lambda_2)t} .$$

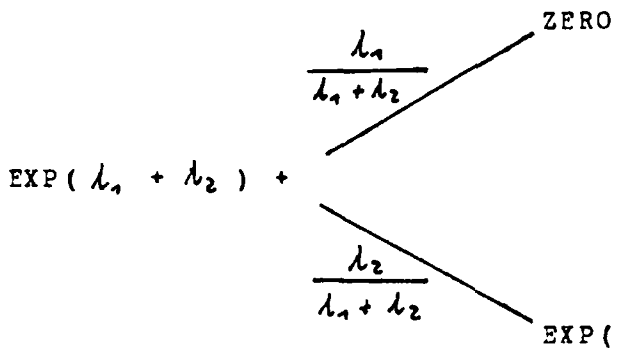
To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

-either both components survive

-or component 2 fails and its spare survives.

If one component fails, in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the time it will be component 1, which means that the system will not survive; in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the time the failing component will be component 2.

This can graphically be represented as



Using the MIX-notation the survival function then is

$$\text{EXP}(\lambda_1 + \lambda_2) + \text{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}\text{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2}\text{EXP}(\lambda_1+\lambda_2)\ ]$$

$$= \text{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{ZERO} + \text{EXP}(\lambda_1 + \lambda_2)),$$
$$\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\ ]$$

$$= \text{MIX}[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1 + \lambda_2)),$$
$$\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\ ].$$

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

$$\bar{F}(t) = \frac{\lambda_1}{\lambda_1+\lambda_2}e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-(\lambda_1+\lambda_2)t} + \right.$$

$$\int_0^t e^{-(\lambda_1+\lambda_2)(t-s)} (\lambda_1+\lambda_2)\, e^{-(\lambda_1+\lambda_2)s}\, ds)$$

$$= (1 + \lambda_2 t)\, e^{-(\lambda_1+\lambda_2)t} .$$

This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. Two-out-of-Three System

As a last example in this chapter, we will look at a Two-out-of- Three system.

Consider a three component system, whose components have constant failure rates λ_1, λ_2 and λ_3 respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system .

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$\begin{aligned}
\bar{F}(t) = {} & P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t) \\
& + P(T_2 > t \wedge T_3 > t) \\
& - P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)) \\
& - P((T_1 > t \wedge T_2 > t) \wedge (T_2 > t \wedge T_3 > t)) \\
& - P((T_1 > t \wedge T_3 > t) \wedge (T_2 > t \wedge T_3 > t))
\end{aligned}$$

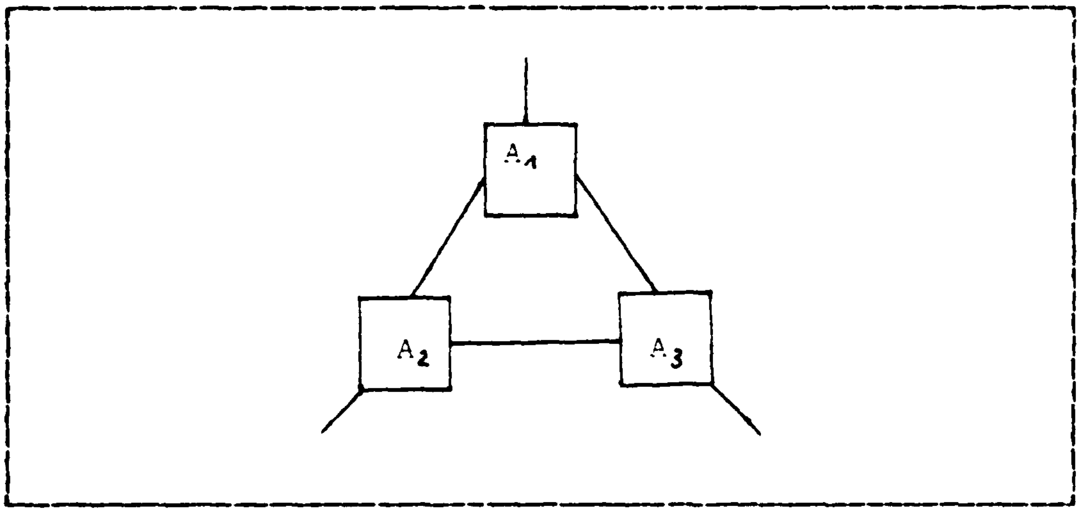


Figure 6: Two-out-of-Three System

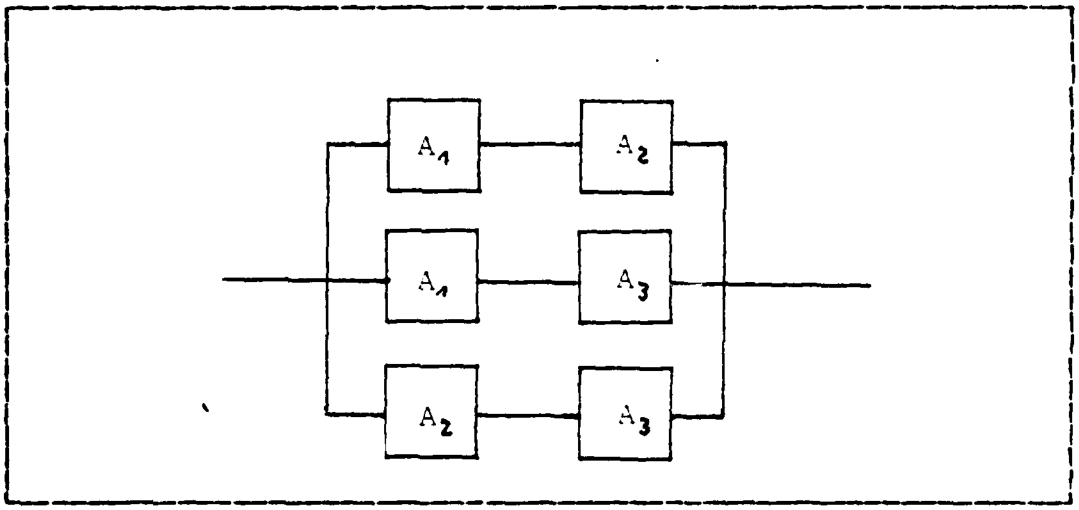


Figure 7: Two-out-of-Three System

$$+ P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)$$
$$\wedge P(T_2 > t \wedge T_3 > t)).$$

Thus

$$\bar{F}(t) = P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t)$$
$$+ P(T_2 > t \wedge T_3 > t)$$
$$- 3P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$
$$+ P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$

Therefore, and assuming independence of the components,

$$\bar{F}(t) = P(T_1 > t)\, P(T_2 > t) + P(T_1 > T)\, P(T_3 > t)$$
$$+ P(T_2 > t)\, P(T_3 > t)$$
$$- 3P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$
$$+ P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$

$$= P(T_1 > t)\, P(T_2 > t) + P(T_1 > t)\, P(T_3 > t)$$
$$+ P(T_2 > t)\, P(T_3 > t)$$
$$- 2P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$
$$= e^{-(\lambda_1+\lambda_2)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t}$$
$$- 2e^{-(\lambda_1+\lambda_2+\lambda_3)t} .$$

Now let us consider all the possible ways, in which the system can survive:

- either all components survive
- or component 1 fails and component 2 and 3 survive
- or component 2 fails and component 1 and 3 survive

- or component 3 fails and component 1 and 2 survive.

If a component fails and the other two survive, in $\frac{\lambda_i}{\lambda_1+\lambda_2+\lambda_3}$ fraction of the time it will be component i , i = 1,2,3 .

This can graphically be represented as

$$\text{EXP}(\lambda_1+\lambda_2+\lambda_3) + \begin{cases} \xrightarrow{\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}} \text{EXP}(\lambda_2+\lambda_3) \\ \xrightarrow{\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}} \text{EXP}(\lambda_1+\lambda_3) \\ \xrightarrow{\frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}} \text{EXP}(\lambda_1+\lambda_2) \end{cases}$$

The shorthand notation then is

$$\begin{aligned} \text{EXP}(\lambda_1+\lambda_2+\lambda_3) + \text{MIX}[& \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_2+\lambda_3), \\ & \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_1+\lambda_3), \\ & \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_1+\lambda_2)], \end{aligned}$$

$$\begin{aligned} = \text{MIX}[& \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_2+\lambda_3) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3)), \\ & \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_1+\lambda_3) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3)), \\ & \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3))]. \end{aligned}$$

Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

$$F(t) = \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_2+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_2+\lambda_3)e^{-(\lambda_2+\lambda_3)s}\,ds\Big]$$

$$+ \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_3)e^{-(\lambda_1+\lambda_3)s}\,ds\Big]$$

$$+ \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_2)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)s}\,ds\Big]$$

$$= e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2)t} - 2e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The result again proves that the shorthand notation indeed represents the survival function of the system.

IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", OA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) - Curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact, that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.

Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.

V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal

computer using a general algorithm for the convolution of any number of exponential random variables.

APPENDIX A

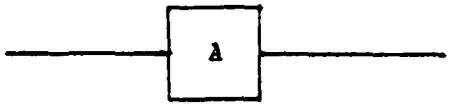
CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.

System:



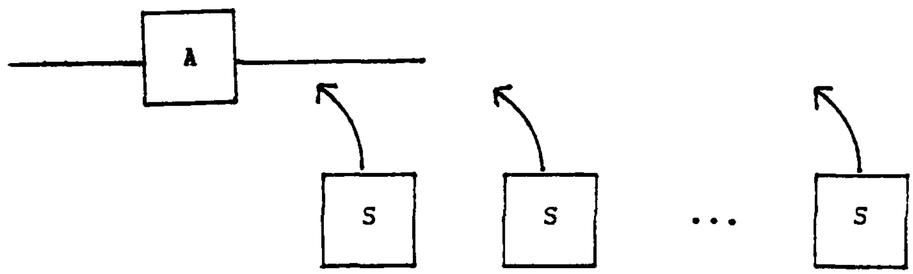
Shorthand: $EXP(\lambda)$

Survival Function: $\bar{F}(t) = e^{-\lambda t}$

Description:

A single active component with constant failure rate λ.

System:



Shorthand: EXP(λ) + EXP(λ) + ... + EXP(λ)

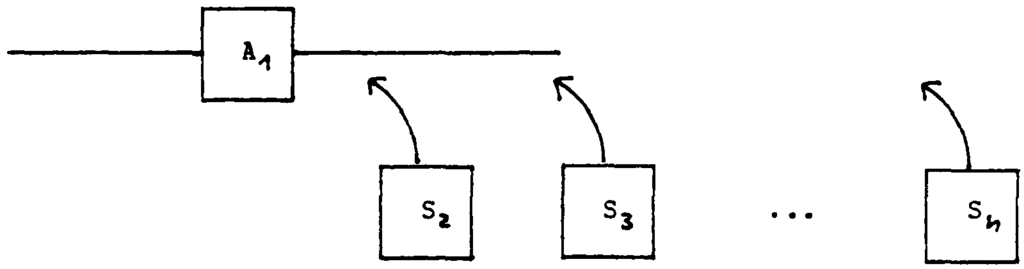
Survival Function: $\bar{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!} \right) e^{-\lambda t}$

$$= \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

Description:

A single active component with constant failure rate is supported by n-1 identical spares.

System:



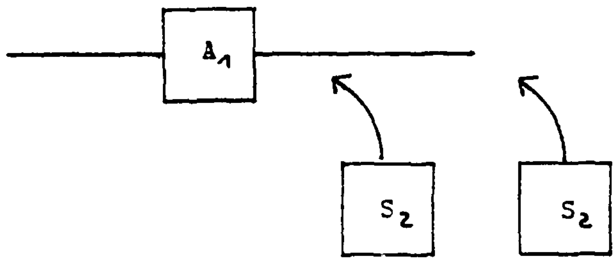
Shorthand: $\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_2) + \ldots + \mathrm{EXP}(\lambda_n)$

Survival Function: $$\bar{F}(t) = \sum_{i=1}^{n} \left(\prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i} e^{-\lambda_i t} \right)$$

Description:

A single active component with constant failure rate is supported by n-1 spares. The active component and the spares have all constant, but dissimilar failure rates.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\overline{F}(t) = Ae^{-\lambda_1 t} + (B + Ct)\, e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^{\,2}}{(\lambda_2 - \lambda_1)^2}$

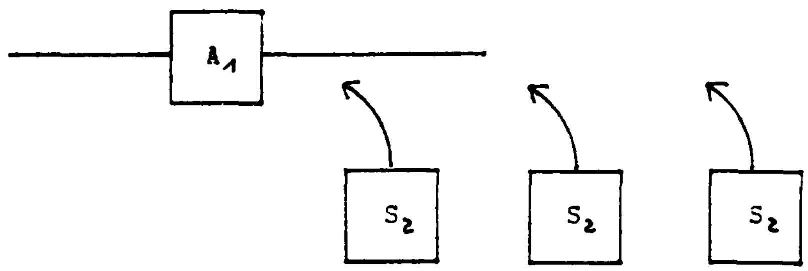
$B = 1 - A$

$C = \dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

Description:

A single active component with constant failure rate λ_1 is supported by two spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct + Dt^2)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^{\,3}}{(\lambda_2 - \lambda_1)^3}$

$B = 1 - A$

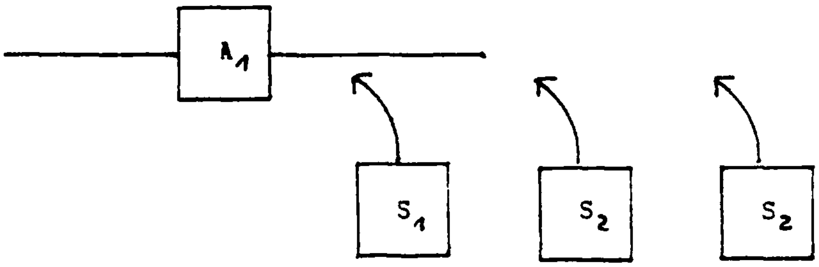
$C = \lambda_2 - \dfrac{\lambda_2^{\,3}}{(\lambda_1 - \lambda_2)^2}$

$D = \dfrac{\lambda_1 \lambda_2^{\,2}}{2\,(\lambda_1 - \lambda_2)}$

Description:

A single active component with constant failure rate λ_1 is supported by three spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + Dt)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^3 - 3\lambda_2^2 \lambda_1}{(\lambda_2 - \lambda_1)^3}$

$B = \dfrac{\lambda_1 \lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

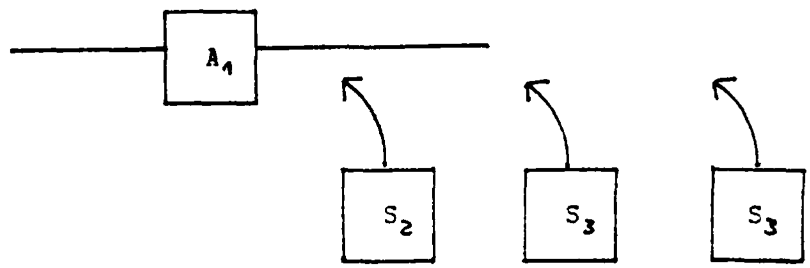
$C = 1 - A$

$D = \dfrac{\lambda_1^2 \lambda_2}{(\lambda_1 - \lambda_2)^2}$

Description:

A single active component with constant failure rate λ_1 is supported by one identical spare and two spares with dissimilar, constant failure rate λ_2 .

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3) + EXP(\lambda_3)$

Survival Function: $\overline{F}(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t} + (C + Dt)e^{-\lambda_3 t}$

where
$$A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)^2}$$

$$B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)^2}$$

$$C = \frac{\lambda_1 \lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} + \frac{\lambda_1\lambda_2\lambda_3}{(\lambda_1-\lambda_2)}\left(\frac{1}{(\lambda_1-\lambda_3)^2} - \frac{1}{(\lambda_2-\lambda_3)^2}\right)$$

$$D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}$$

Description:

A single active component with constant failure rate λ_1 has three spares. One spare has constant failure rate λ_2, two spares are identical with constant failure rate λ_3.

APPENDIX B

USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

$$EXP(\lambda) + EXP(\lambda) + \ldots + EXP(\lambda)$$

or

$$EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n).$$

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7 .

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.B. . Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.

All results will be printed, if the TI-59 is connected to a TI PC-100A or TI PC-100C printer.

PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)

2. Press 2nd C' to initialize.

3. Enter n, the number of exponential terms to be convolved, in the display and press A.

4. Enter time t and press B.

5. Enter λ_i and press C . When all failure rates are the same, enter λ only once.

6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.

   b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

PROGRAM 1 : Sample Problems

1. Find the survival probability of a parallel system

   (compare Chapter III.A.2)

   a) λ = .3 , t = 7 , n = 2

   b) Shorthand notation:

      EXP(.6) + EXP(.3)

   c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
7	t	B	7
.6	2λ	C	.3
.3	λ	C	.3
	$\bar{F}(t)$	B'	.2299172797

calculation takes 13 seconds

2. Find the survival probability of a standby-system with dissimilar components (compare Chapter III.A.3) .

a) $\lambda_1 = .4$, $\lambda_2 = .5$, $t = 6$, $n = 2$

b) Shorthand notation:

EXP(.4) + EXP(.5)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
6	t	B	6
.4	λ_1	C	.4
.5	λ_2	C	.5
	$\overline{F}(t)$	B'	.254441493

calculation takes 13 seconds

3. Find the survival probability of a standby-system with one active component and four similar spares.

a) $\lambda = .3$, $t = 7$, $n = 5$

b) Shorthand notation:

EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
5	n	A	0
7	t	B	7
.3	λ	C	.3
	$\overline{F}(t)$	A'	.9378738848

calculation takes 9 seconds

PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.

   Re-Partition (enter 2 in the display, press 2nd Op 17).

   Read in all four sides of the magnetic card.

2. Press 2nd C' to initialize.

3. Enter n, the number of exponential terms to be convolved, in the display and press A.

4. Enter time t and press B.

5. Enter λ_i and press C (n entries) .

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

6. To find the survival probability of the system press E.

PROGRAM 2, CASE I : Sample Problems

(1) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .3$, $\lambda_2 = .4$, $t = 7$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
3	n	A	0
7	t	B	7
.3	λ_1	C	.3
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\bar{F}(t)$	E	.5363473866

calculation takes 14 seconds

(2) Shorthand notation

EXP(λ_1) + EXP(λ_2) + EXP(λ_2) + EXP(λ_2)

Sample values : λ_1 = .2 , λ_2 = .4 , t = 3

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
3	t	B	3
.2	λ_1	C	.2
.4	λ_2	C	.4
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\bar{F}(t)$	E	.9809746099

calculation takes 20 seconds

(3) Shorthand notation

EXP(λ_1) + EXP(λ_1) + EXP(λ_2) + EXP(λ_2)

Sample values : λ_1 = .4 , λ_2 = .3 , t = 5

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
5	t	B	5
.4	λ_1	C	.4
.4	λ_1	C	.4
.3	λ_2	C	.3
.3	λ_2	C	.3
	$\bar{F}(t)$	E	.9029040721

calculation takes 20 seconds

(4) Shorthand notation

EXP(λ_1) + EXP(λ_2) + EXP(λ_3) + EXP(λ_3)

Sample values : λ_1 = .1 , λ_2 = .3 , λ_3 = .5 ,

t = 10

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
10	t	B	10
.1	λ_1	C	.1
.3	λ_2	C	.3
.5	λ_3	C	.5
.5	λ_3	C	.5
	$\bar{F}(t)$	E	.7312684703

calculation takes 25 seconds

PROGRAM 2 : Procedure

CASE II : to solve problems of the kind, that were introduced in Chapter III.B. .

1. Derive the system's shorthand notation. Find either the
   - graphical representation or
   - the MIX-notation .
2. Use any library module.
   Re-Partition (enter 2 in the display, press 2nd Op 17).
   Read in all four sides of the magnetic card.
3. Press 2nd C' to initialize.
4. Enter time t and press B.
5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.
   a) Enter n, the number of exponential terms to be convolved, in the display and press A.
   b) Enter λ_i and press C.
      REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.
   c) Enter p_i, the weight in the ith path, and press D.
   d) To find the part of the system's survival probability, that is contributed by the ith path, press E.

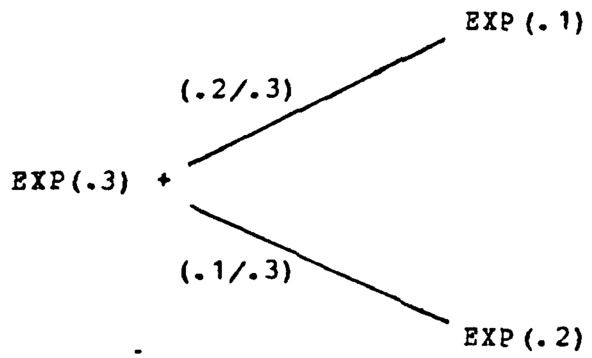
6. To find the survival probability of the system press 2nd E'.

PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system with dissimilar failure rates (compare Chapter III.B.1).

   a) $\lambda_1 = .1$, $\lambda_2 = .2$, $t = 2$

   b) Shorthand notation



$\overline{F}(t)$ = MIX[(.2/.3) (EXP(.1) + EXP(.3),

(.1/.3) (EXP(.2) + EXP(.3)].

c)

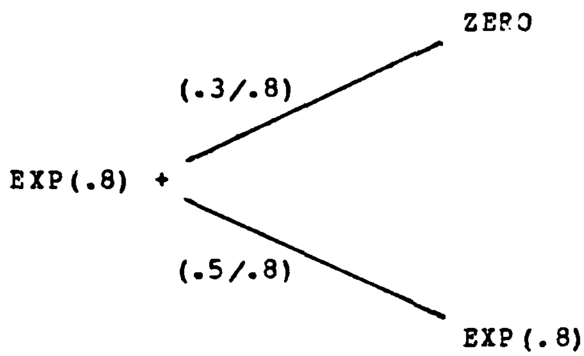
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
2	t	B	2
2	n_1	A	0
.1	λ_1	C	.1
.3	$\lambda_1 + \lambda_2$	C	.3
(.2/.3)	p_1	D	.6666666667
		E	.635793541
2	n_2	A	0
.2	λ_2	C	.2
.3	$\lambda_1 + \lambda_2$	C	.3
(.1/.3)	p_2	D	.3333333333
		E	.304445622
	$\bar{F}(t)$	E'	.940239163

2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2 .

a) $\lambda_1 = .3$, $\lambda_2 = .5$, $t = 7$

b) Shorthand notation



$\bar{F}(t)$ = MIX[(.3/.8)(EXP(.8),

(.5/.8)(EXP(.8) + EXP(.8)].

c)

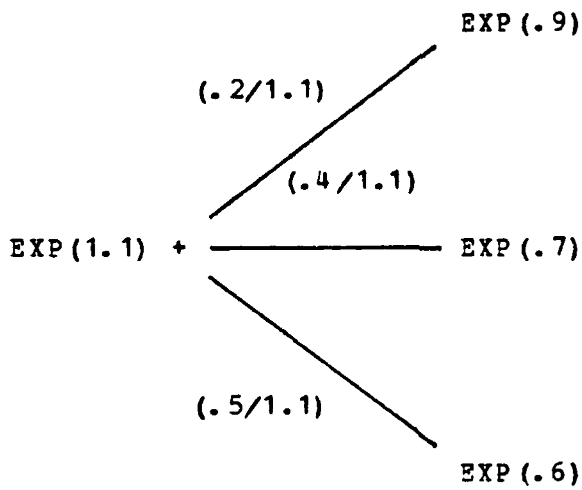
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
7	t	B	7
1	n_1	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
(.3/.8)	p_1	D	.375
		E	.0013866989
2	n_2	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
.8	$\lambda_1 + \lambda_2$	C	.8
(.5/.8)	p_2	D	.625
		E	.0152536878
	$\overline{F}(t)$	E'	.0166403867

3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3 .

a) $\lambda_1 = .2$, $\lambda_2 = .4$, $\lambda_3 = .5$, $t = 9$

b) Shorthand notation



$$\bar{F}(t) = MIX[\ (.2/1.1)\,(\ EXP(.9) + EXP(1.1)),$$
$$(.4/1.1)\,(\ EXP(.7) + EXP(1.1)),$$
$$(.5/1.1)\,(\ EXP(.6) + EXP(1.1))\ ].$$

c)

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
9	t	B	9
2	n_1	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.9	$\lambda_2 + \lambda_3$	C	.9
(.2/1.1)	p_1	D	.1818181818
		E	.0002624871
2	n_2	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.7	$\lambda_1 + \lambda_3$	C	.7
(.4/1.1)	p_2	D	.3636363636
		E	.0018043754
2	n_3	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.6	$\lambda_1 + \lambda_2$	C	.6
(.5/1.1)	p_3	D	.4545454545
		E	.0044892129
	$\overline{F}(t)$	E'	.0065560755

COMPUTER LISTINGS

PROGRAM 1

```
000  91 R/S
001  76 LBL
002  18 C'
003  29 CP
004  25 CLR
005  47 CMS
006  81 RST
007  76 LBL
008  11  A
009  42 STO
010  00  00
011  75  -
012  01  1
013  95  =
014  42 STO
015  09  09
016  29 CP
017  02  2
018  00  0
019  42 STO
020  08  08
021  01  1
022  42 STO
023  17  17
024  00  0
025  42 STO
026  18  18
027  91 R/S
028  76 LBL
029  12  B
030  42 STO
031  01  01
032  91 R/S
033  76 LBL
034  13  C
035  72 ST*
036  08  08
037  69 OP
038  28  28
039  91 R/S
040  76 LBL
041  16 A'
042  43 RCL
043  20  20
044  65  ×
045  43 RCL
046  01  01
047  95  =
048  42 STO
049  02  02
050  00  0
051  01  1
052  42 STO
053  04  04
054  42 STO
055  05  05
056  42 STO
057  18  18
058  76 LBL
059  28 LOG
060  29 CP
061  43 RCL
062  09  09
063  67  EQ
064  29 CP
065  43 RCL
066  02  02
067  45 Y^X
068  43 RCL
069  05  05
070  55  ÷
071  43 RCL
072  04  04
073  95  =
074  44 SUM
075  18  18
076  69 OP
077  39  39
078  69 OP
079  25  25
080  43 RCL
081  05  05
082  49 PRD
083  04  04
084  61 GTO
085  28 LOG
086  76 LBL
087  29 CP
088  43 RCL
089  02  02
090  94 +/-
091  22 INV
092  23 LNX
093  65  ×
094  71 SBR
095  47 CMS
096  95  =
097  99 PRT
098  91 R/S
099  76 LBL
100  17 B'
101  02  2
102  00  0
103  42 STO
104  08  08
105  76 LBL
106  39 COS
107  02  2
108  00  0
109  42 STO
110  06  06
111  01  1
112  42 STO
113  16  16
114  76 LBL
115  38 SIN
116  29 CP
117  73 RC*
118  06  06
119  67  EQ
```

PROGRAM 1 continued

```
120  36 PGM
121  73 RC*
122  08  08
123  32 X↕T
124  73 RC*
125  06  06
126  22 INV
127  67  EQ
128  30 TAN
129  69 DP
130  26  26
131  61 GTD
132  38 SIN
133  76 LBL
134  30 TAN
135  49 PRD
136  16  16
137  75  -
138  73 RC*
139  08  08
140  95  =
141  35 1/X
142  49 PRD
143  16  16
144  43 RCL
145  00  00
146  32 X↕T
147  43 RCL
148  06  06
149  75  -
150  01  1
151  09  9
152  95  =
153  77  GE
154  36 PGM
155  69 DP
156  26  26
157  61 GTD
158  38 SIN
159  76 LBL
160  36 PGM
161  73 RC*
162  08  08
163  94 +/-
164  65  ×
165  43 RCL
166  01  01
167  95  =
168  22 INV
169  23 LNX
170  65  ×
171  43 RCL
172  16  16
173  95  =
174  44 SUM
175  18  18
176  43 RCL
177  00  00
178  32 X↕T
179  43 RCL
180  08  08
181  75  -
182  01  1
183  09  9
184  95  =
185  67  EQ
186  37 P/R
187  69 DP
188  28  28
189  61 GTD
190  39 CDS
191  76 LBL
192  37 P/R
193  71 SBR
194  47 CMS
195  99 PRT
196  91 R/S
197  76 LBL
198  47 CMS
199  43 RCL
200  18  18
201  92 RTN
```

LABEL ADRESSES

```
002  18 C'
008  11  A
029  12  B
034  13  C
041  16 A'
059  28 LOG
087  29 CP
100  17 B'
106  39 CDS
115  38 SIN
134  30 TAN
160  36 PGM
192  37 P/R
198  47 CMS
```

PROGRAM 2

000	86	STF
001	00	00
002	61	GTO
003	15	E
004	76	LBL
005	18	C'
006	29	CP
007	25	CLR
008	47	CMS
009	91	R/S
010	76	LBL
011	11	A
012	42	STO
013	00	00
014	75	-
015	01	1
016	95	=
017	42	STO
018	09	09
019	29	CP
020	01	1
021	00	0
022	42	STO
023	08	08
024	01	1
025	42	STO
026	17	17
027	00	0
028	42	STO
029	18	18
030	91	R/S
031	76	LBL
032	12	B
033	42	STO
034	01	01
035	91	R/S
036	76	LBL
037	13	C
038	72	ST*
039	08	08
040	69	OP
041	28	28
042	91	R/S
043	76	LBL
044	14	D
045	49	PRD
046	17	17
047	91	R/S
048	76	LBL
049	16	A'
050	43	RCL
051	10	10
052	65	×
053	43	RCL
054	01	01
055	95	=
056	42	STO
057	02	02
058	00	0
059	01	1
060	42	STO
061	04	04
062	42	STO
063	05	05
064	42	STO
065	18	18
066	76	LBL
067	28	LOG
068	29	CP
069	43	RCL
070	09	09
071	67	EQ
072	29	CP
073	43	RCL
074	02	02
075	45	YX
076	43	RCL
077	05	05
078	55	÷
079	43	RCL
080	04	04
081	95	=
082	44	SUM
083	18	18
084	69	OP
085	39	39
086	69	OP
087	25	25
088	43	RCL
089	05	05
090	49	PRD
091	04	04
092	61	GTO
093	28	LOG
094	76	LBL
095	29	CP
096	43	RCL
097	02	02
098	94	+/-
099	22	INV
100	23	LNX
101	65	×
102	71	SBR
103	47	CMS
104	95	=
105	91	R/S
106	76	LBL
107	17	B'
108	01	1
109	00	0
110	42	STO
111	08	08
112	76	LBL
113	39	COS
114	01	1
115	00	0
116	42	STO
117	06	06
118	01	1
119	42	STO

PROGRAM 2 continued

```
120  16  16
121  76  LBL
122  38  SIN
123  29  CP
124  73  RC*
125  06  06
126  67  EQ
127  36  PGM
128  73  RC*
129  08  08
130  32  X:T
131  73  RC*
132  06  06
133  22  INV
134  67  EQ
135  30  TAN
136  69  OP
137  26  26
138  61  GTO
139  38  SIN
140  76  LBL
141  30  TAN
142  49  PRD
143  16  16
144  75  -
145  73  RC*
146  08  08
147  95  =
148  35  1/X
149  49  PRD
150  16  16
151  43  RCL
152  00  00
153  32  X:T
154  43  RCL
155  06  06
156  75  -
157  09  9
158  95  =
159  77  GE
160  36  PGM
161  69  OP
162  26  26
163  61  GTO
164  38  SIN
165  76  LBL
166  36  PGM
167  73  RC*
168  08  08
169  94  +/-
170  65  ×
171  43  RCL
172  01  01
173  95  =
174  22  INV
175  23  LNX
176  65  ×
177  43  RCL
178  16  16
179  95  =
180  44  SUM
181  18  18
182  43  RCL
183  00  00
184  32  X:T
185  43  RCL
186  08  08
187  75  -
188  09  9
189  95  =
190  67  EQ
191  37  P/R
192  69  OP
193  28  28
194  61  GTO
195  39  COS
196  76  LBL
197  37  P/R
198  71  SBR
199  47  CMS
200  91  R/S
201  76  LBL
202  47  CMS
203  43  RCL
204  18  18
205  65  ×
206  43  RCL
207  17  17
208  95  =
209  99  PRT
210  44  SUM
211  19  19
212  92  RTN
213  76  LBL
214  22  INV
215  43  RCL
216  11  11
217  33  X²
218  55  ÷
219  53  (
220  43  RCL
221  11  11
222  75  -
223  43  RCL
224  10  10
225  54  )
226  33  X²
227  95  =
228  42  STO
229  07  07
230  65  ×
231  53  (
232  43  RCL
233  10  10
234  94  +/-
235  65  ×
236  43  RCL
237  01  01
238  54  )
239  22  INV
```

PROGRAM 2 continued

```
240  23  LNX
241  95  =
242  42  STO
243  18  18
244  43  RCL
245  07  07
246  94  +/-
247  85  +
248  01  1
249  85  +
250  43  RCL
251  10  10
252  65  ×
253  43  RCL
254  11  11
255  55  ÷
256  53  (
257  43  RCL
258  10  10
259  75  -
260  43  RCL
261  11  11
262  54  )
263  65  ×
264  43  RCL
265  01  01
266  95  =
267  65  ×
268  53  (
269  43  RCL
270  11  11
271  94  +/-
272  65  ×
273  43  RCL
274  01  01
275  54  )
276  22  INV
277  23  LNX
278  95  =
279  44  SUM
280  18  18
281  71  SBR
282  47  CMS
283  91  R/S
284  76  LBL
285  23  LNX
286  43  RCL
287  11  11
288  45  YX
289  03  3
290  55  ÷
291  53  (
292  43  RCL
293  11  11
294  75  -
295  43  RCL
296  10  10
297  54  )
298  45  YX
299  03  3
300  95  =
301  42  STO
302  07  07
303  65  ×
304  53  (
305  43  RCL
306  10  10
307  94  +/-
308  65  ×
309  43  RCL
310  01  01
311  54  )
312  22  INV
313  23  LNX
314  85  +
315  53  (
316  43  RCL
317  07  07
318  94  +/-
319  85  +
320  01  1
321  85  +
322  53  (
323  43  RCL
324  11  11
325  75  -
326  43  RCL
327  11  11
328  45  YX
329  03  3
330  55  ÷
331  53  (
332  43  RCL
333  10  10
334  75  -
335  43  RCL
336  11  11
337  54  )
338  33  X²
339  54  )
340  65  ×
341  43  RCL
342  01  01
343  85  +
344  43  RCL
345  10  10
346  65  ×
347  43  RCL
348  11  11
349  33  X²
350  55  ÷
351  53  (
352  02  2
353  65  ×
354  53  (
355  43  RCL
356  10  10
357  75  -
358  43  RCL
359  11  11
```

PROGRAM 2 continued

360	54	)
361	54	)
362	65	×
363	43	RCL
364	01	01
365	33	X^2
366	54	)
367	65	×
368	53	(
369	43	RCL
370	11	11
371	94	+/-
372	65	×
373	43	RCL
374	01	01
375	54	)
376	22	INV
377	23	LNX
378	95	=
379	44	SUM
380	18	18
381	71	SBR
382	47	CMS
383	91	R/S
384	76	LBL
385	24	CE
386	43	RCL
387	12	12
388	65	×
389	33	X^2
390	75	-
391	03	3
392	65	×
393	43	RCL
394	12	12
395	33	X^2
396	65	×
397	43	RCL
398	10	10
399	95	=
400	55	÷
401	53	(
402	53	(
403	43	RCL
404	12	12
405	75	-
406	43	RCL
407	10	10
408	54	)
409	65	×
410	33	X^2
411	54	)
412	95	=
413	42	STO
414	07	07
415	85	+
416	43	RCL
417	10	10
418	65	×
419	43	RCL
420	12	12
421	33	X^2
422	55	÷
423	53	(
424	43	RCL
425	12	12
426	75	-
427	43	RCL
428	10	10
429	54	)
430	33	X^2
431	65	×
432	43	RCL
433	01	01
434	95	=
435	65	×
436	53	(
437	43	RCL
438	10	10
439	94	+/-
440	65	×
441	43	RCL
442	01	01
443	54	)
444	22	INV
445	23	LNX
446	95	=
447	42	STO
448	18	18
449	53	(
450	43	RCL
451	07	07
452	94	+/-
453	85	+
454	01	1
455	85	+
456	43	RCL
457	10	10
458	33	X^2
459	65	×
460	43	RCL
461	12	12
462	55	÷
463	53	(
464	43	RCL
465	10	10
466	75	-
467	43	RCL
468	12	12
469	54	)
470	33	X^2
471	65	×
472	43	RCL
473	01	01
474	54	)
475	65	×
476	53	(
477	43	RCL
478	12	12
479	94	+/-

PROGRAM 2 continued

480	65	×
481	43	RCL
482	01	01
483	54	)
484	22	INV
485	23	LNX
486	95	=
487	44	SUM
488	18	18
489	71	SBR
490	47	CMS
491	91	R/S
492	76	LBL
493	25	CLR
494	43	RCL
495	11	11
496	65	×
497	43	RCL
498	12	12
499	33	X^2
500	95	=
501	55	÷
502	53	(
503	43	RCL
504	11	11
505	75	-
506	43	RCL
507	10	10
508	54	)
509	55	÷
510	53	(
511	43	RCL
512	12	12
513	75	-
514	43	RCL
515	10	10
516	54	)
517	33	X^2
518	95	=
519	65	×
520	53	(
521	43	RCL
522	10	10
523	94	+/-
524	65	×
525	43	RCL
526	01	01
527	54	)
528	22	INV
529	23	LNX
530	95	=
531	42	STO
532	18	18
533	43	RCL
534	10	10
535	65	×
536	43	RCL
537	12	12
538	33	X^2
539	55	÷
540	53	(
541	43	RCL
542	10	10
543	75	-
544	43	RCL
545	11	11
546	54	)
547	55	÷
548	53	(
549	43	RCL
550	12	12
551	75	-
552	43	RCL
553	11	11
554	54	)
555	33	X^2
556	95	=
557	65	×
558	53	(
559	43	RCL
560	11	11
561	94	+/-
562	65	×
563	43	RCL
564	01	01
565	54	)
566	22	INV
567	23	LNX
568	95	=
569	44	SUM
570	18	18
571	43	RCL
572	10	10
573	65	×
574	43	RCL
575	11	11
576	55	÷
577	53	(
578	43	RCL
579	10	10
580	75	-
581	43	RCL
582	12	12
583	54	)
584	55	÷
585	53	(
586	43	RCL
587	11	11
588	75	-
589	43	RCL
590	12	12
591	54	)
592	95	=
593	42	STO
594	07	07
595	55	÷
596	43	RCL
597	10	10
598	65	×
599	43	RCL

PROGRAM 2 continued

600	11	11
601	65	×
602	43	RCL
603	12	12
604	55	÷
605	53	(
606	43	RCL
607	10	10
608	75	-
609	43	RCL
610	11	11
611	54	)
612	65	×
613	53	(
614	53	(
615	43	RCL
616	10	10
617	75	-
618	43	RCL
619	12	12
620	54	)
621	33	X^2
622	35	1/X
623	75	-
624	53	(
625	43	RCL
626	11	11
627	75	-
628	43	RCL
629	12	12
630	54	)
631	33	X^2
632	35	1/X
633	54	)
634	95	=
635	85	+
636	43	RCL
637	07	07
638	65	×
639	43	RCL
640	12	12
641	65	×
642	43	RCL
643	01	01
644	95	=
645	65	×
646	53	(
647	43	RCL
648	12	12
649	94	+/-
650	65	×
651	43	RCL
652	01	01
653	54	)
654	22	INV
655	23	LNX
656	95	=
657	44	SUM
658	18	18
659	71	SBR
660	47	CMS
661	91	R/S
662	76	LBL
663	10	E'
664	98	ADV
665	43	RCL
666	19	19
667	99	PRT
668	91	R/S
669	76	LBL
670	15	E
671	87	IFF
672	00	00
673	60	DEG
674	81	RST
675	76	LBL
676	60	DEG
677	22	INV
678	86	STF
679	00	00
680	86	STF
681	40	IND
682	00	00
683	00	0
684	42	STO
685	07	07
686	43	RCL
687	11	11
688	32	X:T
689	87	IFF
690	01	01
691	16	A'
692	87	IFF
693	02	02
694	48	EXC
695	87	IFF
696	03	03
697	49	PRD
698	87	IFF
699	.04	04
700	50	I×I
701	76	LBL
702	48	EXC
703	43	RCL
704	10	10
705	67	EQ
706	16	A'
707	61	GTO
708	17	B'
709	76	LBL
710	49	PRD
711	43	RCL
712	10	10
713	67	EQ
714	16	A'
715	43	RCL
716	12	12
717	67	EQ
718	22	INV
719	61	GTO

PROGRAM 2 continued

720	17	B'
721	76	LBL
722	50	I×I
723	43	RCL
724	10	10
725	22	INV
726	86	STF
727	40	IND
728	00	00
729	22	INV
730	67	EQ
731	57	ENG
732	01	1
733	42	STD
734	07	07
735	76	LBL
736	57	ENG
737	43	RCL
738	12	12
739	22	INV
740	67	EQ
741	58	FIX
742	02	2
743	44	SUM
744	07	07
745	86	STF
746	40	IND
747	07	07
748	87	IFF
749	03	03
750	59	INT
751	22	INV
752	86	STF
753	40	IND
754	07	07
755	76	LBL
756	58	FIX
757	43	RCL
758	12	12
759	32	X↕T

760	43	RCL
761	13	13
762	22	INV
763	67	EQ
764	59	INT
765	04	4
766	44	SUM
767	07	07
768	76	LBL
769	59	INT
770	86	STF
771	40	IND
772	07	07
773	87	IFF
774	00	00
775	17	B'
776	87	IFF
777	03	03
778	16	A'
779	87	IFF
780	04	04
781	25	CLR
782	87	IFF
783	05	05
784	24	CE
785	87	IFF
786	06	06
787	23	LNX
788	91	R/S

LABEL ADRESSES

005	18	C'
011	11	A
032	12	B
037	13	C
044	14	D
049	16	A'
067	28	LOG
095	29	CP
107	17	B'
113	39	COS
122	38	SIN
141	30	TAN
166	36	PGM
197	37	P/R
202	47	CMS
214	22	INV
285	23	LNX
385	24	CE
494	25	CLR
664	10	E'
671	15	E
677	60	DEG
700	48	EXC
708	49	PRD
720	50	I×I
734	57	ENG
754	58	FIX
767	59	INT

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